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SOME PROPERTIES OF THE DYNAMICS OF A PUNCH–LAYER SYSTEM: AN ENERGY ANALYSIS†

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The dynamics of unsteady interactions in a system consisting of a punch and an elastic layer is studied. It is shown that the system can theoretically accumulate as large an oscillation energy as desired. The mechanism of releasing the stored energy is also studied. The efficiency of the system as an oscillation energy accumulator is studied numerically. The mechanism by which this energy is released when the mass of the punch changes instantaneously is analysed.

We know that the oscillations of an ideally elastic body of finite dimensions can be characterized by a denumerable system of natural resonance frequencies and corresponding eigenfunctions. Under a harmonic load, the frequency of which coincides with a resonance oscillation frequency of the body, the oscillation amplitudes increase with time without limit. As opposed to a body of finite dimensions, an ideally elastic layer has no natural modes of oscillation. However, it has been established [1] that a finite number of natural frequencies with corresponding natural modes of oscillation can exist in a punch–layer system. This effect, referred to as an isolated resonance [1], was studied and generalized in [2, 3] and became known as the B-resonance. In a punch–layer system subject to a harmonic resonance load the oscillation amplitudes increase without limit, as in the case of a body of finite dimensions. As a consequence, an oscillation energy as large as desired can theoretically be "accumulated" in the system. The distribution density of this energy in the layer is determined by the eigenfunctions of the punch–layer system, which, as has been shown in [4], decay exponentially with distance from the punch. The energy of the natural oscillations can therefore be considered localized in the neighbourhood of the punch. The energy of the natural oscillations can therefore be considered localized in the neighbourhood of the punch.

Suppose that in a steady natural mode of oscillation the parameters of the punch–layer system are altered, while the total energy remains unchanged. The system undergoes a transition into a natural mode of oscillation with different frequency, the energy connected with the new mode of oscillation being localized near the punch. The difference between the steady oscillation energies before and after the change of parameters is the energy radiated to infinity. In what follows it will be proved that the entire stored oscillation energy can be reradiated to infinity as a high-energy impulse. The values of the displacement amplitudes in this impulse can be much larger than those generated by a source of the same power without the storage of energy in the system.

As a result, a punch–layer system can exhibit the properties of both an ideal body of finite dimensions, namely, the presence of natural modes of oscillation, and an unbounded continuum, namely, energy radiation to infinity. The processes of energy storage and the radiation of a high-energy impulse in the system resemble the principles governing pulsed lasers.

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We shall investigate those relationships between the parameters that give rise to B-resonances in the system and study the processes of energy storage and reradiation.

1. We shall prove theorems that hold for a large class of continuous media of finite as well as infinite dimensions. These theorems can be used in the study of a punch–layer system to find the exact boundaries of the domain of existence of a B-resonance in the case of oscillations of a massive punch normal to the surface of the layer. In addition, the results can significantly simplify the analysis of the energy balance of the system.

Let Ω be a domain occupied by an elastic continuum with an arbitrary linear equation of state

$$\sigma_{jk} = C_{jkmn}\varepsilon_{mn},\ \varepsilon_{jk} = 1/2(u_{j,k} + u_{k,j}) \tag{1.1}$$

where σ_{jk}, ε_{jk}, u_j are the components of the stress tensor, the strain tensor, and the displacement vector at a point of the medium, respectively, and C_{jkmn} are the components of the elastic modulus tensor having the well-known symmetry and positive definiteness properties [5]. We assume that C_{jkmn} are sufficiently smooth functions of the coordinates. The index notation traditionally adopted in continuous medium mechanics is used in (1.1) and in what follows.

We consider steady oscillations of a continuum subjected to an external periodic load (the time dependence being expressed by the multiplier $e^{-i\omega t}$). On separating the variables, we write the equations of motion in the form

$$\sigma_{jk,k} + \omega^2\rho u_j + \rho F_j = 0 \tag{1.2}$$

where ρ is the density of the medium and F_j is the amplitude function of volume forces.

Theorem 1. If the stress and strain fields satisfy the Gauss–Ostrogradskii conditions in an arbitrary domain Ω with smooth boundary S, then

$$\frac{1}{2}\int_S \sigma_{jk}l_k\bar{u}_j dS + \frac{1}{2}\int_\Omega \rho F_j\bar{u}_j d\Omega = \Pi(\omega) - K(\omega)$$

$$\Pi(\omega) = \frac{1}{2}\int_\Omega \sigma_{jk}\bar{\varepsilon}_{jk} d\Omega,\ K(\omega) = \frac{1}{2}\omega^2\int_\Omega \rho u_j\bar{u}_j d\Omega$$

where l_j is the outward normal vector to S.

This theorem is an analogue of Clapeyron's theorem [5] and can be proved in a similar way. The overbar denotes a compled conjugate quantity. In the case of real-valued fields, $\Pi(\omega)$ and $K(\omega)$ are the maximum values of the potential and kinetic energy of the elastic medium, respectively.

We will now consider the problem of an absolutely rigid body (the punch) in contact with an elastic continuum, the latter occupying a domain Ω with boundary S. The punch undergoes harmonic oscillations of amplitude W_0 and direction $\mathbf{n}$, the contact domain S_0 between the punch and the medium remaining unchanged during the oscillations, which means that the punch and the medium do not separate. Homogeneous conditions of the first, second, or third kind are given on S away from the die. In the following results we can restrict ourselves, without loss of generality, to conditions of the first and second kind. We assume that there are no mass forces. Let us write down the boundary conditions on $S = S_0 \cup S_1 \cup S_2$

$$\mathbf{u} = \begin{cases} W_0\mathbf{n} & \text{on } S_0 \\ 0 & \text{on } S_2; \end{cases} \quad \sigma_{jk}l_k = 0 \quad \text{on } S_1 \tag{1.3}$$

Note that for unbounded domains the boundary-value problem (1.1)–(1.3) must be supplemented by radiation conditions. In addition, to ensure that the mixed problem is well posed, we will impose restrictions on the behaviour of the solution on the boundary of the contact domain S_0 [4].

Theorem 1 implies that

$$W_0 \int_{S_0} \sigma_{jk} l_j n_k dS = 2\{\Pi(\omega) - K(\omega)\}$$

We set $r_n(\omega) = 2\{\Pi(\omega) - K(\omega)\}/W_0^2$. It is easily seen that r_n is the dynamic rigidity of the medium, i.e. its reaction to unit amplitude oscillations of the die. By the linearity of the problem under consideration, $\Pi(\omega)$ and $K(\omega)$ are proportional to W_0^2, which means that r_n is independent of W_0.

Corollary 1. Under the hypotheses of Theorem 1, $r_n(0) = \Pi(0) \geqslant 0$.

Note that for bodies of finite dimensions as well as for some semi-bounded continua (for example, a layer or a half-plane) $\Pi(\omega) \to 0$ as $\omega \to 0$ if the stresses are given on the boundary outside the contact domain S_0 (conditions of the first kind).

Theorem 2. If the solution of the boundary-value problem (1.1)–(1.3) in an arbitrary domain Ω (bounded or unboun ded) satisfied the hypotheses of Theorem 1 and is real-valued and analytic in ω, then $dr_n/d\omega = -4K(\omega)/\omega W_0^2$.

Due to the symmetry of the components of the elastic modulus tensor

$$\partial(\sigma_{jk}\varepsilon_{jk})/\partial\omega = 2\sigma_{jk}\partial\varepsilon_{jk}/\partial\omega$$

Hence, using Theorem 1, we find that

$$W_0^2 \frac{dr_n}{d\omega} = 2\int_\Omega \sigma_{jk} \frac{\partial\varepsilon_{jk}}{\partial\omega} d\Omega - 2\omega^2 \int_\Omega \rho u_j \frac{\partial u_j}{\partial\omega} d\Omega - 2\omega \int_\Omega \rho u_j u_j d\Omega$$

Applying the Gauss–Ostrogradskii theorem to the right-hand side, we get

$$W_0^2 \frac{dr_n}{d\omega} = 2\int_\Omega (\sigma_{jk,k} + \omega^2 \rho u_j) \frac{\partial u_j}{d\omega} d\Omega - \frac{4K(\omega)}{\omega} + \int_S \sigma_{jk} l_k \frac{\partial u_j}{d\omega} dS$$

The first term on the right-hand side vanishes by virtue of (1.2). The surface integral also vanishes, because the stresses on S_1 are equal to zero by (1.3), and, by the analyticity of the solution in ω, we have $\partial\mathbf{u}/\partial\omega = 0$ on S_2 and $\partial\mathbf{un}/\partial\omega = 0$ on S_0.

Corollary 2. Under the hypotheses of Theorem 2, $dr_n/d\omega < 0$.

Remarks. Theorem 2 holds when conditions of the third kind are given on the boundary S or some part of it.

All the results presented above hold for any elliptic wave equation with the appropriate modifications in the expressions for the potential and kinetic energies and the boundary conditions.

2. Consider the oscillations of a massive punch normal to the surface of an elastic layer of thickness H. We will assume that the elastic moduli C_{jkmn} depend on the x_3-coordinate normal to the planes constituting the boundaries of the layer. The boundary $x_3 = 0$ of the layer is rigidly attached to a non-deformable support, i.e. S_2 coincides with the plane $x_3 = 0$. On the boundary $x_3 = H$ the stresses are equal to zero everywhere outside the contact domain between the punch and the layer. Therefore, we assume that $\mathbf{n} = \mathbf{e}_3$ in (1.3) (one of the unit vectors of the Cartesian system of coordinates) and the outward normal vector $\mathbf{l}$ to S_1 is equal to $\mathbf{e}_3$. We shall write the equation for the oscillations of the punch in the form

$$-m_0\omega^2 W_0 = R_3(\omega), R_3(\omega) = -\int_{S_0} \sigma_{33} dS \tag{2.1}$$

where m_0 is the mass of the punch.

Assuming that there are no volume forces in (1.2), we shall study the eigenvalues of the boundary-value problem (1.1)–(1.3), (2.1). We shall state a number of properties of the boundary-value problem (1.1)–(1.3), which will be used in our study. Their proofs are given in [4].

Property 1. There exists $\omega_1 > 0$ such that for $\omega \in (-\omega_1, +\omega_1)$ the energy solution is real-valued and decays exponentially at infinity.

Property 2. the solution is analytic in ω everywhere on the real axis, except for a denumerable $\pm\omega_k$ $(k = 1, 2, \dots)$.

Property 3. For all positive $\omega > \omega_1$ $\operatorname{Im} R_3(\omega) \leqslant 0$ and $R_3(-\omega) = \overline{R}_3(\omega)$.

We note that the sign of the imaginary part $R_3(\omega)$ is determined by the time dependence expressed by the factor $e^{-i\omega t}$, which has been chosen at an earlier stage. We set $m_* = \lim r_3(\omega)/\omega^2$ as $\omega \to \omega_1$, where $r^3(\omega) = r_n(\omega)$ $(\mathbf{n} = \mathbf{e}_3)$. Since the boundary-value problem (1.1)–(1.3) is linear, $R_3(\omega) = W_0 r_3(\omega)$. Substituting this relationship into (2.1), we obtain the characteristic equations from which to determine the eigenvalues

$$r_3(\omega) - m_0\omega^2 = 0 \tag{2.2}$$

Theorem 3. For any $m_0 > m_*$ the boundary-value problem (1.1)–(1.3), (2.1) has two eigenvalues $\pm\omega_0$ of multiplicity one belonging to the interval $(-\omega_1, +\omega_1)$.

The proof of this theorem follows from the results presented in the previous section and the aforesaid properties of the solution of the boundary-value problem (1.1)–(1.3) for a layer.

The existence of B-resonances in a punch–layer system for sufficiently large values of the mass of the punch was proved in [1]. Theorem 3 establishes the precise boundary of the domain of existence of a B-resonance in a system in which the oscillations of the punch are normal to the surface of the layer.

In Fig. 1 we show the domains of existence of B-resonances for an isotropic elastic layer (the plane problem) for various values of Poisson's ratio ϑ using the axes M^{-1} $(M = m_0/(\rho H^2))$ and $L = \lg(a/H)$. The layer interacts with a strip-shaped punch of linear mass m_0 and transverse dimension $2a$. The domains of existence of an isolated resonance lie below the curves. As can be seen in Fig. 1, the boundary of the domain depends strongly on Poisson's ratio of the material.

Computations indicate that for a punch rigidly attached to the layer as well as for a punch in frictionless contact with the layer, the domains of existence of B-resonances are practically the same. All numerical results were obtained for problems involving frictionless contact.

The computations were carried out using a software package for solving a large class of mixed dynamic problems for semi-bounded domains. the solution algorithm is based on a modification of the boundary element method.

Corollary 3. The expression

$$E = \tfrac{1}{2} m_0 \omega_0^2 W_0^2 - \tfrac{1}{4} \omega_0 W_0^2 r_3'(\omega_0) \tag{2.3}$$

holds for the total energy of characteristic oscillations of the punch–layer system.

This expression follows from the fact that for sinusiodal (due the solution being real-valued) natural oscillations of the system the potential energy vanishes simultaneously at all points of the elastic continuum (i.e. the total oscillation energy of the system is equal to its maximum kinetic energy) and from the law of conservation of energy (no radiation at infinity).

In Fig. 2 we show the specific energy of natural oscillations of the punch–layer system. Here the specific energy is understood as the total energy of the system per unit amplitude of oscillation of the punch. The coordinate axes in the horizontal plane are the same as in Fig. 1, the specific energy being measured along the vertical axis. As the boundary of the domain of existence of a B-resonance is approached, the specific energy tends to infinity. The surface in

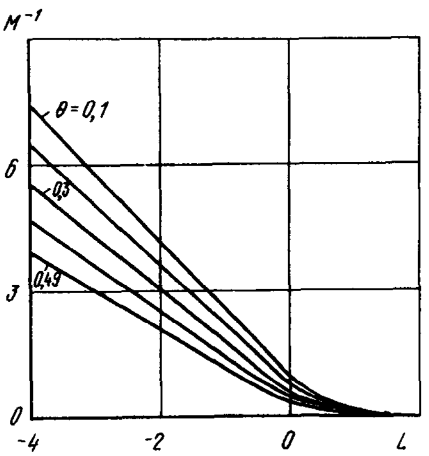

Fig. 1.

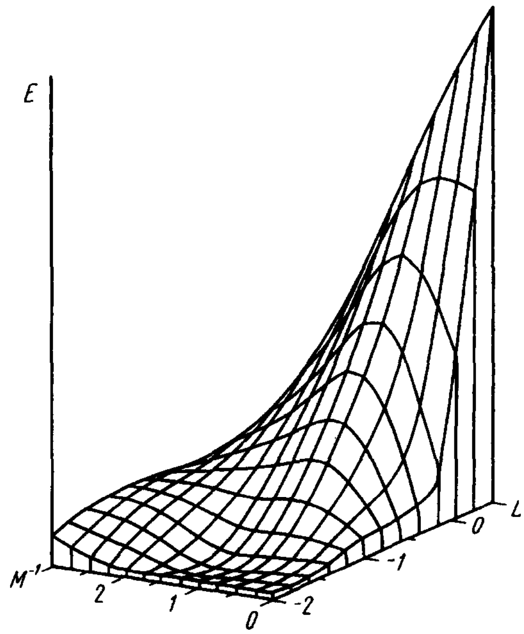

Fig. 2.

Fig. 2 represents the variation of the specific energy in the domain of existence of a B-resonance and its growth as the boundary is approached. In Fig. 2, in a very narrow neighbourhood of the boundary the energy gradient becomes so large for small values of a/H that it cannot be represented graphically.

3. We shall investigate the process of energy accumulation in the system. Let the system be in a state of rest for $t<0$. At $t=0$ a load $P(t)$ is applied to the punch, so that the punch begins to advance in the normal direction to the surface of the layer. We shall write down the equations together with the initial and boundary conditions of the problem under consideration

$$\sigma_{jk,k}-\rho\partial^2 u_j/\partial t^2=0 \tag{3.1}$$

$$\sigma_{jk}=C_{jkmn}\varepsilon_{mn},\ \ \varepsilon_{jk}=\tfrac{1}{2}(u_{j,k}+u_{k,j})$$

$$\mathbf{u}=\begin{cases} w(t)\mathbf{e}_3 & \text{on } S_0,\\ 0 & \text{on } S_2;\end{cases}\ \ \sigma_{jk}l_k=0 \text{ on } S_1 \tag{3.2}$$

$$m_0\frac{d^2w}{dt^2}=P(t)+R_3(t),\ \ R_3(t)=-\int\limits_{S_0}\sigma_{33}dS \tag{3.3}$$

$$\mathbf{u}=0,\ \partial\mathbf{u}/\partial t=0;\ w=0,\ dw/dt=0,\ t=0 \tag{3.4}$$

We apply a Fourier transformation with respect to the initial boundary-value problem (3.1)–(3.4). We shall use the same symbols for the transforms as for the original functions, specifying whether they depend on the time or frequency, if necessary. For example

$$P(\omega)=\int\limits_0^\infty P(t)\ \exp(i\omega t)\ dt,\ \operatorname{Im}\omega=\delta>0$$

On applying the transformation, we get

$$w(t)=-(2\pi)^{-1}\int\limits_{-\infty+i\delta}^{+\infty+i\delta}P(\omega)[m\omega^2-r_3(\omega)]^{-1}\exp(-i\omega t)d\omega \tag{3.5}$$

The work done by P (or the energy received by the system) is

$$A(t)=\int_0^t P(t)\frac{dw}{dt}dt$$

If the force acts during a bounded time interval, the work can be written as

$$A(t)=-\frac{1}{2\pi i}\int_{-\infty+i\delta}^{+\infty+i\delta} P^*(\omega)P(\omega)[m_0\omega^2-r_3(\omega)]^{-1}\omega\,d\omega \tag{3.6}$$

$$P^*(\omega)=\int_0^\infty P(t)\ \exp(-i\omega t)\,dt$$

for long times t.

We shall assume that the punch–layer system is such that the hypothesis of Theorem 3 holds for the corresponding boundary-value problem, i.e. there is a B-resonance $\pm\omega_0$ in the system. By taking the limit as $\delta\to+0$ in (3.6), we get

$$A=\omega_0\left|P(\omega_0)\right|^2[2m_0\omega_0-dr_3(\omega_0)/d\omega]^{-1}+A_\infty \tag{3.7}$$

$$A_\infty=-(2\pi i)^{-1}\,\mathrm{V.p.}\int_{-\infty}^{+\infty}\left|P(\omega)\right|^2[m_0\omega^2-r_3(\omega)]^{-1}\omega\,d\omega$$

With the same assumptions, we find from (3.5) the amplitude

$$W_0=2\left|P(\omega_0)\right|[2m_0\omega_0-dr_3(\omega_0)/d\omega]^{-1} \tag{3.8}$$

of steady oscillations of the system.

Taking (2.3) and (3.8) into account, it can be shown that the first term in (3.7) is the total energy of steady oscillations of amplitude W_0 given by (3.8). The second term A_∞ is therefore the energy radiated to infinity as the system subjected to the force $P(t)$ undergoes a transition from a state of rest to a steady mode of oscillation. Note once more that the force acts during a bounded time interval.

Taking Property 3 into account, we obtain the representation

$$A_\infty=-\pi^{-1}\int_{\omega_1}^{\infty}\mathrm{Im}\,r_3(\omega)\left|P(\omega)\right|^2[m_0\omega^2-r_3(\omega)]^{-2}\omega\,d\omega$$

We observe that if there is no isolated resonance in the system, then $A=A_\infty$, i.e. all the energy supplied to the system is radiated to infinity.

Let us consider the process of energy accumulation in the system in the case when the external force $P(t)$ has the form

$$P(t)=\begin{cases}P_0\sin(\omega_0 t), & 0<t<2\pi N/\omega_0\\ 0, & t<0, t>2\pi N/\omega_0\end{cases}$$

i.e. a periodic load is applied, the frequency of which is equal to a natural frequency of the system, the duration of the interaction being characterized by the number of periods N. Then the amplitude W_0 of natural oscillations given by (3.8) is proportional to N, so that the energy E of natural oscillations is proportional to N^2. It follows that if the amplitude of the external force is bounded, then the energy of natural oscillations can become as large as desired.

The following normalized values are presented below for a system consisting of an elastic layer ($\nu=0.3$) and a strip-shaped punch ($a/H=1$; $M=1$) for various values of N: the energy A supplied to the system, the accumulated energy E, the energy A_∞ lost due to radiation, and index $\eta = (E/A) \times 100\%$ measuring the efficiency of energy accumulation

N	1	2	3	4	10
A	2.623	10.480	23.578	41.913	261.940
E	2.619	10.478	23.575	41.910	261.940
$A_\infty \times 10^3$	4	3	3	2	4
$(100-\eta) \times 10^3$	153	27	14	5	1

The absolute value of the energy loss due to radiation is seen to be practically independent of the number of periods N, while the index η approaches 100% as N increases. The results illustrating the dependence of A, E, A_∞ and η on M are presented below in the case when $N = 1$

M	0.65	0.70	1.00	2.00	100.0
A	2.581	2.592	2.623	2.629	2.686
E	2.491	2.565	2.619	2.629	2.686
$A_\infty \times 10^3$	89	27	4	≈ 0	≈ 0
$(100-\eta) \times 10^3$	470	37	153	12	≈ 0

Calculations show that when the mass decreases and becomes close to M_* (for the given system $M_* = 0.633$), the efficiency index η decreases slightly because of increased energy loss due to radiation. However, the amount of accumulated oscillation energy is practically independent of M for a wide range of masses. Therefore the punch–layer system turns out to be a highly efficient store of oscillation energy.

4. We shall consider in detail the process of radiation of accumulated oscillation energy. Suppose that natural oscillations of frequency ω_0 (ω_0 being a root of Eq. (2.2)) are generated in the punch–layer system by a force $P(t)$. For a long time after the force $P(t)$ is switched off the oscillations of the system can be regarded as steady, i.e. the time dependence of the displacements of any point of the system can be expressed by the multiplier $\cos(\omega_0 t)$. Moreover, the energy of steady natural oscillations of the system can be expressed in the form (2.3), where the amplitude multiplier W_0 is determined by the specific form of $P(t)$ (see (3.8)).

Suppose that the mass of the punch changes instantanoeusly at the instant when the velocity of the punch is zero (we shall assume that this instant is $t = 0$). Since the total energy of the system at this instant is equal to the potential energy of the elastic deformation of the continuum, the change of mass does not alter the total energy of the system. However, the characteristic frequency of the system will be altered. We shall denote by M_0 the new value of the mass of the die and by ω_0 the corresponding natural frequency. In the mathematical formulation of the problem Eqs (3.1) and boundary conditions (3.2) are supplemented by the equation of motion of the punch

$$M_0 \frac{d^2 w}{dt^2} = R_3(t), \quad R_3(t) = \int_{S_0} \sigma_{33} dS \tag{4.1}$$

and the initial conditions

$$\mathbf{u} = \mathbf{u}_0, \partial \mathbf{u} / \partial t = 0; \quad w = W_0, \quad dw/dt = 0, \quad t = 0 \tag{4.2}$$

where $\mathbf{u}_0$ is the natural solution of the boundary-value problem (1.1)–(1.3), (2.1) corresponding to the natural frequency ω_0.

We shall solve problem (3.1), (3.2), (4.1) and (4.2) by the method of integral transformations. We write the following expressions for the displacements of the punch

$$w(t)=W_0\cos(\omega_0 t)-(2\pi)^{-1}W_0(M_0\omega_0^2-r_3(\omega_0))\int_{-\infty+i\delta}^{+\infty+i\delta}\frac{-i\omega}{\omega_0^2-\omega^2}\,\frac{\exp(-i\omega t)}{M_0\omega^2-r_3(\omega)}\,d\omega$$

By applying Jordan's lemma to this expression and letting $t\to\infty$, we get

$$w(t)=2W_0\frac{\Omega_0}{\omega_0^2-\Omega_0^2}\,\frac{M_0\omega_0^2-r_3(\omega_0)}{2M_0\Omega_0-r_3'(\Omega_0)}\cos(\Omega_0 t)+o(1) \tag{4.3}$$

Taking (2.3) and (4.3) into account, we can represent the energy of steady oscillations of the system with altered mass in the form

$$E_0=\frac{\Omega_0^3}{(\omega_0^2-\Omega_0^2)^2}\,\frac{[M_0\omega_0^2-r_3(\omega_0)]^2}{2M_0\Omega_0-r_3'(\Omega_0)}\,W_0^2$$

Theorem 4. Let the punch–layer system be in a steady state of oscillation. Any change in the mass of the punch not accompanied by a change of the total energy of the system will result in the radiation of some energy to infinity.

It follows from the law of conservation of energy that the energy radiated to infinity is equal to $E_\infty=E-E_0$. Using elementary algebraic transformations and taking Corollary 2 into account, it can be shown that $E_\infty>0$ for any mass change.

Corollary 4. In the frequency range $\omega\in(0,\ \omega_1)$ the maximum value $\Pi(\omega)$ of the potential energy of oscillations of the punch of amplitude W_0 exceeds the potential energy of a static elastic deformation for the same displacement W_0 of the punch.

Remark. If the new value of the mass $M_0<m_*$, then the entire oscillation energy of the system will be radiated to infinity. As follows from Theorem 3, for such a value of m the system has no natural frequencies, so that $E_0=0$.

Figure 3 shows the radiated energy E_∞/E as a function of the change of mass of the punch. Curves 1, 2, and 3 were obtained for various values of the linear mass of a strip-shaped punch ($m_*/m_0=0.5,\ 0.9,\ 0.99$) of width $a/H=1$ interacting with an isotropic elastic layer ($\nu=0.3$). These masses correspond to the characteristic frequencies $\omega_0/\omega_1=1.2,\ 1.517,\ 1.57$. To represent the results in a convenient way and consistently with the previous graphs, the ratio E_∞/E is shown as a function of m_*/M_0. For such a choice of the variable it is obvious that the curves touch the axis of abscissae at the points 0.5, 0.9 and 0.99 ($E_\infty/E=0$), respectively. One can see in Fig. 3 that an increase in mass ($M_0>m_0$) as well as a decrease are accompanied by energy radiation to infinity. It is, however, clear that the energy radiated as a result of a decrease in mass can be much larger than that due to an increase in mass.

The behaviour of curves 1 and 2 indicates that the energy of the system is close to that of a static system in the range of frequencies ω between 0 and $0.96\omega_1$, i.e. there is virtually no energy radiation to infinity (less than 5%) when the punch is abruply stopped ($M_0=\infty$). The frequency range ($0.96\omega_1$, ω_1) is therefore a rather narrow transition zone, in which the elastic layer is transformed from a quasistatic to a dynamic state, namely, for $\omega>\omega_1$ the oscillations in the layer are no longer sinusoidal and homogeneous propagating waves appear. This explains the sharp increase in the energy radiated to infinity ($E_\infty/E\to 1$ as $M_0\to m_*$) and the large gradients of curves 1–3 in a small neighbourhood to the left of the point $m_*/M_0=1$, which corresponds to ω_1. Large gradients appeared before in Fig. 2.

We shall illustrate the radiation process by the problem of an anti-plane displacement of a layer by a strip-shaped punch. Let the radiation of energy be caused by "complete separation" of the punch and the layer. The problem can be reduced by the superposition method to solving the initial boundary-value problem for the wave equation in a strip with homogeneous initial conditions and boundary conditions of the form

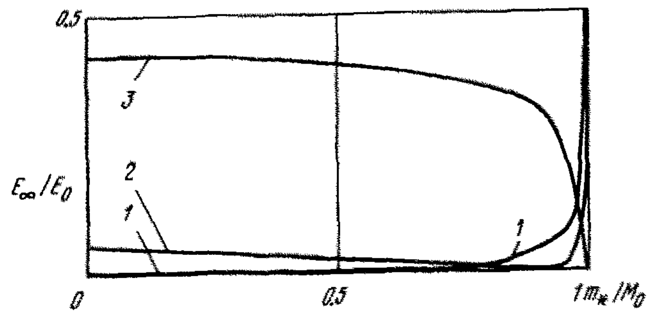

Fig. 3.

$$u=0,\ x_3=0;\quad \frac{G\partial u}{\partial x_3}=\begin{cases} q(x_1)\cos(\omega_0 t)h(t), |x_1|< a; \\ 0,\ |x_1|> a; \end{cases}\quad x_3=H$$

where $q(x_1)$ is the amplitude of the contact stresses in the problem of the natural oscillations of the system with frequency ω_0 when the displacement amplitude of the punch equals W_0, $h(t)$ being the Heaviside function. We shall write the solution of this problem in the form

$$u(x_1,x_3,\tau)=i(4\pi)^{-2}\int_{-\infty}^{\infty} Q(\alpha,\kappa_0)\exp(-i\alpha x_1)d\alpha \int_{-\infty+i\delta}^{+\infty+i\delta} \kappa(\kappa^2-\kappa_0^2)^{-1}\times$$
$$\times \mathrm{sh}[(\alpha^2-\kappa^2)^{1/2}x_3][(\alpha^2-\kappa^2)^{1/2}\mathrm{ch}(\alpha^2-\kappa^2)^{1/2}]^{-1}\exp(-i\kappa\tau)d\kappa$$

where $\kappa=\omega H/c$, and $\tau=ct/H$ (c is the velocity of propagation of the transverse wave), the linear dimensions and displacements are normalized to H, and $Q(\alpha,\ \kappa_0)$ denotes the Fourier transform of $q(x_1)/G$ with respect to x_1. For large x_1 the asymptotic expressions for the displacement u has the form

$$u(x_1,x_3,\tau)=\frac{(1-x_t^2)^{1/2}}{\tau}\sum_{n=1}^{\infty}(-1)^n \sin\left[\pi\left(n-\frac{1}{2}x_3\right)\right](2n-1)^{1/2}\times$$
$$\times Q\left(\frac{\pi(n-1/2)x_t}{(1-x_t^2)^{1/2}},\kappa_0\right)\cos\left[\pi\left(n-\frac{1}{2}\right)\tau(1-x_t^2)^{1/2}+\frac{\pi}{4}\right]\times$$
$$\times\left(\pi^2\left(n-\frac{1}{2}\right)^2-\kappa_0^2(1-x_t^2)\right)^{-1},\quad x_t=\frac{x_1}{\tau}<1$$

The time dependence of the displacement of a point on the free surface of the layer ($x_3=1$) computed from the above asymptotic formula is shown in Fig. 4, in which the envelopes of the radiated oscillating signal are shown ($x_1=100$, $x_1=500$). The dimensionless time τ is measured along the horizontal axis and the dimensionless displacement along the vertical axis. The oscillation frequency shown in a small part of the forefront of the impulse (Fig. 5) is equal to the frequency computed by the reflection method [6]. We observe that the impulse becomes longer and the maximum displacement in the impulse decreases as the x_1-coordinate of the observation point increases.

5. The energy radiated to infinity is transmitted by an unsteady impulse in which both the head and tail perturbation fronts can be distinguished far away from the punch, since in a steady mode of oscillation the displacement field decays exponentially at infinity. It follows from (4.3) that the displacements in the impulse are proportional to W_0. As has been observed before, amplitudes of the natural oscillations as large as desired can be attained during the process of energy accumulation by a system subjected to external interactions of bounded

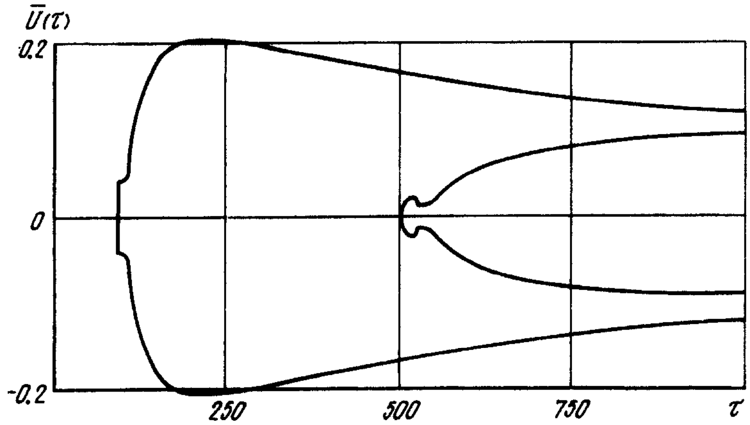

Fig. 4.

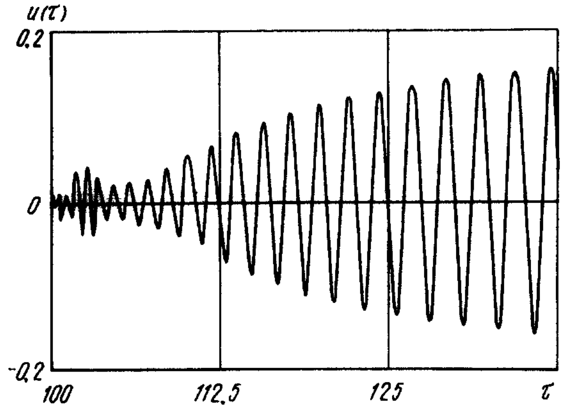

Fig. 5.

amplitude. Considering energy accumulation and radiation as a single process, we conclude that the displacements in the impulse can be much larger than those generated by a source of the same power without energy accumulation in the system. One can therfore talk of high-energy impulse generation.

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